CHAPTER 5

Arguments good and bad: an introduction to philosophical logic for practitioners

Chapter contents

Achilles joyfully exclaimed, as he ran the pencil into its sheath. 'And at last we've got to the end of this ideal race-course! Now that you accept *A* and *B* and *C* and *D, of course* you accept *Z*.'

'Do I?' said the Tortoise innocently. 'Let's make that quite clear. I accept *A* and *B* and *C* and *D*. Suppose I *still* refuse to accept *Z*?'

'Then Logic would take you by the throat, and *force* you to do it!' Achilles triumphantly replied.

What The Tortoise Said To Achilles
Lewis Carroll, originally published in *Mind*
(1895, No. 4, pp. 278–280)

We will return to Lewis Carroll's account of the debate between the Tortoise and Achilles about the effectiveness of logic a little later in this chapter. However, Achilles's suggestion that Logic might take one by the throat and force one to a conclusion is a striking expression of the power it holds on our conception of reason and rationality. At the same time, formal logic is often thought to be a subject only for specialists. In this chapter we aim to give an outline of some of the tools available for argument and some of the general issues raised by them.

Two aims of the chapter

Like Chapter 3 (on psychopathology), this chapter has two main aims. The first is the counterpart of the corresponding first aim of Chapter 3, i.e. to give non-philosophers a 'feel' for the work of professional philosophers. Just as medical and psychology students learn the details of psychopathology, so most undergraduates on philosophy degrees include logic; and just as psychiatrists and clinical psychologists have to be able use current diagnostic classifications, so professional philosophers have to be able to use logic.

Fig. 5.1 Lewis Carroll

The second aim of this chapter is in a sense the opposite of the corresponding aim of Chapter 3. In setting out the details of descriptive psychopathology, we aimed to get across to philosophers that if they want to work in this area, there is a good deal of detail with which they have to be prepared to get up to speed—detail derived not just from professional but also lay (narrative) sources. The message was, psychopathology is not as easy as it may seem!

Here, the message is that logic is not as *difficult* as it may seem. Of course, like most subjects, advanced logic is difficult. However, the basics are relatively easy to grasp. Logic, as you will see, is a set of tools for developing and testing arguments. It helps to frame sound arguments, and to spot arguments that are unsound. So, the second aim of this chapter is to start you thinking in a critical philosophical way as a contribution to the improved 'thinking skills' that are one of main outputs from engagement with philosophy in mental health (Chapter 1). In this chapter, therefore, we provide an introduction to some of the main principles of logic, and the terminology you are likely to come across, together with suggestions for further reading.

How to read this chapter . . .

Unless you are interested in logic for its own sake, there is no need to struggle with the details of this chapter. The intention is not to turn practitioners into 'mini-logicians' (most philosophers do a full course on Logic and Philosophy of Logic in their first degree). It is rather to enable you to become familiar with the terminology of logic, and, in the process, to start to develop your own skills of argument.

. . . and what you should get out of it

1. *Terminology*. By the end of the chapter you should have developed a degree of familiarity with a number of important terms that philosophers pick up by absorption during a first degree (e.g. 'entails', 'paradox', 'syllogism', etc.). All these terms are themselves the subject of detailed philosophical work (we review some of the work on 'implication' later). But for the purposes of this book, you only need to be broadly familiar with them, so that you can 'speak the same language' as philosophers.
2. *Skills of argument*: By the end of the chapter you should also have improved your own skills of argument. As noted above, much of logic is about what makes for a good or bad argument, a valid or fallacious one. These notes aim to outline some of the tools philosophy has come up with to help us here. The importance of an exposure to philosophical logic is not, primarily, to learn the names of this or that fallacy (we list 18 of the more common ones at the end). It is to give you the skills to present good arguments and to spot bad

ones. So this is above all an area where *practice* is essential. Don't just read this chapter passively. Try out the exercises for yourself and invent examples of your own. That said, the aim of this chapter is not to teach you the whole of 'undergraduate logic'. That would take more than a chapter and there are many good introductory textbooks on logic for that purpose. The aim is instead to introduce the kind of tools and way of approaching an argument that is at the heart of logic.

3. *Grasp of the broader issues*. By the end of the chapter you should also have a better understanding of the purpose of logical argument, of its strengths and some of its limitations. The *philosophy of* logic is much less settled than the standard logical tools taught to philosophy students (this is not to say that there is no disagreement at the level of logic research), but the central questions about what underpins inference and so on are important for thinking about reasoning and rationality as a whole: surely of central concern to the philosophy of mental health!

Session 1 An introduction to deductive reasoning and formal logic

Why argue deductively?

Argument is one way of attempting to arrive at the truth. There are other ways, such as careful observation and experimentation but even these are usually combined with argument to establish what they show and their impact on our other beliefs. Argument is thus a key element in any general intellectual project that has truth as its goal. This, however, raises a question: How does the proffering of sentences justify or establish the truth a further sentence? Logic is the name of the codification of how sentences can bear on the truth of other sentences in argument.

An argument is described as having two parts: the *premisses* (usually plural) from which we argue to the *conclusion* (usually singular). The point of an argument is to demonstrate the truth of the conclusion, given the truth of the premisses. This is a *semantic* claim. It is a claim about the extent to which the sentences correctly report the state of the world they purport to describe. (Semantics is the name of the formal study of how words and sentences can refer to and be true of features of the world.)

But if the aim of an argument is to demonstrate the truth of one sentence from the truth of others, if the success of the argument is in question it seems that we cannot rely on this semantic relation holding. To know that the truth of the conclusion follows from the truth of the premisses seems to require that we know both the truth of both premisses and conclusion. But it was the purpose of the argument to establish the truth of the conclusion.

This circularity is broken by appeal not to the content or the particular sentences serving as premisses and conclusion but to the relation of the structure of the sentences. The argument can be characterized not just in semantic terms—in terms of the world-involving claim expressed—but syntactic terms, terms having to do with the structure of sentences. The idea behind 2000 years of the study of deductive logic is that the structure of arguments that establish the truth of their conclusions can be articulated.

Deductive logic thus codifies the relationship between structures of sentences. We will discuss, albeit briefly, three ways of codifying this structure:

- syllogism,
- sentential or propositional logic, and
- predicate logic.

Logical validity, truth, and soundness in syllogisms

Valid and invalid inferences

A good starting point is Aristotle, the first person to write a systematic treatise on logic, and thus the inspiration behind of all systems of modern logic. In his *Topics*, written sometime in the fourth century BC, he gives a number of examples of a form of inference, which is known as the *syllogism*. Every syllogism is a sequence of three propositions such that the first two imply the third, the conclusion. Thus:

1. All men are mortal
   Socrates is mortal
   Therefore Socrates is a man
2. All men are mortal
   Socrates is a man
   Therefore Socrates is mortal

(10 minutes)

Think about these two arguments. One is valid; one not—which is which, and why?

These examples are both syllogisms. They each have two premisses, one of which is general (the major premiss), the other of which is particular (the minor premiss), and a conclusion (third line). They illustrate many of the points we need to cover in this brief introduction to philosophical logic.

Syllogism 1 is *invalid*. It illustrates one of the classical fallacies—that of arguing from the *minor term* ('are mortal') in the major premiss to a conclusion about the *major term* ('all men'). Clearly, the premisses could be true and the conclusion false (e.g. Socrates could be a mortal *dog* consistently with the major and minor premisses).

Syllogism 2 is *valid*. In a valid argument, as we shall see, *if* the premisses are true, then the conclusion *must* be true. This illustrates the compulsion of valid arguments—i.e. if it is true that all men are mortal and that Socrates is a man, then it *has* to be true that Socrates is mortal.

There are in fact very many forms of syllogism, only some of which are valid. The *Columbia Electronic Encyclopedia* gives the following brief account:

> There are three basic types of syllogism: hypothetical, disjunctive, and categorical. The hypothetical syllogism, *modus ponens,* has as its first premiss a conditional hypothesis: *If p then q;* it continues: *p,* therefore *q.* The disjunctive syllogism, *modus tollens,* has as its first premiss a statement of alternatives: *Either p or q;* it continues: not *q,* therefore *p.* The categorical syllogism comprises three categorical propositions, which must be statements of the form *all x are y, no x is y, some x is y,* or *some x is not y.* A categorical syllogism contains precisely three terms: the major term, which is the predicate of the conclusion; the minor term, the subject of the conclusion; and the middle term, which appears in both premisses but not in the conclusion. Thus: *All philosophers are men* (middle term); *all men are mortal*; therefore, *All philosophers* (minor term) *are mortal* (major term). The premisses containing the major and minor terms are named the major and minor premisses, respectively. Aristotle noted five basic rules governing the validity of categorical syllogisms: The middle term must be distributed at least once (a term is said to be distributed when it refers to all members of the denoted class, as in *all x are y* and *no x is y*); a term distributed in the conclusion must be distributed in the premiss in which it occurs; two negative premisses imply no valid conclusion; if one premiss is negative, then the conclusion must be negative; and two affirmatives imply an affirmative.

One of the preoccupations of medieval scholastic philosophy was with the elucidation of the valid forms. But from the above account it is worth asking: How would one determine whether a form of syllogism was valid? What general method is available?

Fortunately, the syllogism is now recognized as a fairly limited form of argument. What it can achieve can be more perspicuously represented using modern logical systems, of which more later. So, thankfully, there is no need to study the many different forms of syllogism!

But even this initial introduction to logical argument raises the question: What makes an argument a good one? What are the qualities or virtues of an argument?

Validity, truth, and soundness

Validity, *truth*, and *soundness* are three of the most important concepts in logic and are related notions. Validity and truth are used in loose and informal senses in everyday assessments of statements and arguments. Soundness is confined to philosophical use.

In common-sense understanding, a valid argument is simply a *good* argument. But what exactly makes an argument good?

EXERCISE 2 (10 minutes)

Go back to syllogism (2) again. Is the conclusion true? On what does its truth or falsity depend? If the premisses were false, could the conclusion be true? If the premisses were true could the conclusion be false?

Now read:

What the Tortoise Said To Achilles, by Lewis Carroll. Originally published in *Mind* (1895) No. 4, 278–280

Link with Reading 5.1

In the light of the first questions, of what would one need to be persuaded in order rationally to be persuaded of the conclusion of the argument discussed: (*Z*) The two sides of this Triangle are equal to each other?

The truth of the premisses and the validity of the argument

Now one response to the first question set would be to say that the sentence 'Socrates is mortal' is true because it reports the fact that Socrates is (or was) mortal. However, in this context the right response turns on the nature of the argument offered. The conclusion is supposed to follow from the premisses asserted in the argument. Thus there are two issues to be resolved:

- Are the premisses true?
- Does the conclusion follow from the premisses?

This double dependence of the conclusion on what has been placed before it is summarized by Carroll's Tortoise thus.

> 'Readers of Euclid will grant, I suppose, that *Z* follows logically from *A* and *B*, so that anyone who accepts *A* and *B* is true, *must* accept *Z* as true?'
>
> 'Undoubtedly! The youngest child in a High School—as soon as High Schools are invented, which will not be till some two thousand years later—will grant *that.*'
>
> 'And if some reader had *not* yet accepted *A* and *B* as true, he might still accept the *Sequence* as a *valid* one, I suppose?'
>
> 'No doubt such a reader might exist. He might say "I accept as true the Hypothetical Proposition that, if *A* and *B* be true, *Z* must be true; but I *don't* accept *A* and *B* as true." Such a reader would do wisely in abandoning Euclid, and taking to football.'
>
> 'And might there not *also* be some reader who would say 'I accept *A* and *B* as true, but I *don't* accept the Hypothetical'?'
>
> 'Certainly there might. *He,* also, had better take to football.'
>
> 'And *neither* of these readers,' the Tortoise continued, 'is *as yet* under any logical necessity to accept *Z* as true?'
>
> 'Quite so,' Achilles assented.
>
> What The Tortoise Said To Achilles, by Lewis Carroll, Originally published in *Mind* (1895, No. 4, pp. 278–280)

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Note two important things: First, the validity of an argument is independent of the truth of its premisses. Secondly, validity functions so as to preserve truth in an argument. Together, the effect is that if the premisses of an argument are true, and the form of the argument is valid, then the conclusion will be true also. If one or more of the premisses of an argument is false, however, then even if its form is valid, we cannot conclude anything at all about the truth of its conclusion. Similarly, if an argument has true premisses but an invalid form may have either a true or false conclusion, just by chance. This is why an argument that has an invalid form cannot be cited as a piece of reasoning from the premisses to the conclusion, in other words the truth of the conclusion does not follow from the truth of the premisses.

(We will return to the Tortoise and Achilles in the next section. But for now it is worth noting that the Tortoise's last claim above is not so innocent as it appears.)

What is 'truth'?

What of 'truth'? How do we know when something is *true*? And, by knowing this, do we thereby have an understanding of the *concept* 'truth'? These are difficult questions (see suggestions for further reading at the end of this chapter). Consider, for example, a long-standing approach to the nature of truth called a 'correspondence theory'. One might aim to clarify the notion of truth by saying that a true sentence, e.g. corresponds to a fact. Aside from the choice of 'truth bearer' (are sentences, propositions, or beliefs the sorts of things that are paradigmatically true or false?) whether this approach can work or not depends, however, on whether the right-hand side can offer clarification. In this statement of the correspondence relation we would need to know independently what sort of thing a *fact* is. And there is reason to believe that our best grasp of what a fact is, is that it is what a true sentence states! In other words, no independent clarification is offered.

A more modest approach takes the connection between a true sentence and the fact it states as marking not an explanation but the limits of the concept. Take an example of what is called the T schema (meaning True schema) (or sometimes the disquotational schema) following the work of the Polish-American mathematician-logician Alfred Tarski in the 1940s:

T schema: 'P' is true if and only if P.

P stands for a sentence. The sentence that 'P' stands for on the left is *mentioned* while on the right it is stripped of its quotation marks and *used* to state a worldly fact. The most famous examples is: 'Snow is white' is true if and only if snow is white.

This says that the sentence on the left, 'snow is white' has a particular property under particular circumstances. It is true if it is a fact that snow is white. The sentence is true if snow is white. Which sentence? 'Snow is white.'

The philosopher of language Donald Davidson calls this a 'snow bound triviality'. It is a particular example of the relation that a correspondence theorist hopes to use to explain truth.

A minimalist or deflationist argues, by contrast, that there is no substantial property of truth that 'true' picks out. As a recent commentator Bernard Weiss puts it: 'Minimalists or deflationists about truth argue that our concept of truth is all but captured via (some version of) the disquotational schema, '*P*' is true iff *P*.' (Weiss, 2002, p. 63). (NB 'iff' just means if and only if.) The underlying idea of such an approach is that there is no substantial explanatory property common to truths. Because 'is true' is a predicate there may be 'no harm' in taking truth to be a property (Dodd, 2000, p. 136). However, there is no prospect of a successful philosophical project to show, e.g. that all truths correspond to facts where 'correspondence' and 'fact' have independent analyses.

The most miminal such theory was put forward by Frank Ramsey. His 'redundancy theory' proposed that since 'is true' served simply as a device for disquotation in T schema instances it could be eliminated without loss (Ramsey, 1927). Truth is not, however, eliminable because of the need sometimes to make indirect or compendious assertions, e.g. in cases where what is said is not known, although it is known to be true. We might want to say, e.g. that everything the Pope said was true. Hence Weiss's (2002) comment that truth is 'all but captured' in the disquotational schema. Nevertheless, this need not commit us to thinking that there is anything more to truth than the transparent property expressed in the equivalence between asserting that *P* and asserting that '*P*' is true (for elaboration see Horwich, 1990).

Fortunately, for present purposes there is no need to offer a substantial analysis of truth, even if that were possible. Logic codifies good arguments on the assumption that we already know enough what truth is.

True premisses plus valid form = a sound argument

We are now in a position to consider the notion of *soundness*. Soundness is the property an argument has if it is both valid and its premisses true. Recall that a valid form of argument is one which, if its premisses are true then it cannot fail to have a true conclusion. The concept of 'soundness' is required because we need some way of referring to such arguments, in virtue of the specific effect of the coincidence of validity and truth. Thus, syllogism (2) is a sound argument. Its premisses are true, its form is valid, and therefore it has a true conclusion.

What is far from clear, however, is what property of an argument serves to guarantee this relation (i.e. of the truth of the premiss(s) to the truth of the conclusion). In order to inquire into this, we need to introduce the idea of *logical operators*.

Summary so far

So far, we have used the example of syllogism to exemplify the idea that deductive logic is a codification of argumentative structure. If an argument is of the right syntactic form then it can possess a further semantic property: if the premisses are true then so is the conclusion. This is the form of a valid argument. Such an argument never leads from truth to falsity.

But validity is not sufficient for the truth of a conclusion. That requires, in addition, that the premisses actually are true. If they are, and if the argument is valid, then it is also a sound argument. Validity of form plus true premisses equals soundness.

But while scholastic philosophy attempted to codify valid forms of argument through syllogisms there is a simpler, more modern approach—propositional or sentential logic—which in turn leads to modern powerful predicate logic devised by Frege. While we will not discuss predicate logic at any length, we will present a brief sketch shortly.

Propositional logic: the logic of simple connectives

Propositional logic and logical operators

Consider the short argument:

> It is raining and the sun is shining
> Therefore, it is raining

This is a valid argument. If the premiss is true then the conclusion must be true. But why?

In this case the answer is obvious. It turns on the structure of the premiss and conclusion. Consider this argument:

> Brix is the best cat in the world and Wittgenstein is the hardest philosopher
>
> Therefore, Brix is the best cat in the world

It shares the same form as the previous argument. If the one is valid then so is the other and in fact both are because of their shared structure which runs as follows:

> P and Q
> Therefore, P

Simple arguments of this form help highlight the way logical form can underpin validity. Take any two basic sentences represented by P and Q, conjoin them with the word 'and' and that gives a complex sentence that can serve as the premiss of an argument from which the first basic sentence can be derived as a valid conclusion.

The branch of logic that deals with this kind of structure is called Propositional or Sentential logic because it takes as its basic building blocks whole sentences or propositions linked together by some basic logical operators.

Sentences and propositions

The word 'proposition' in philosophy refers to the contents of sentences, what can be shared by sentences with the same meaning in different languages, for example. A proposition is what is expressed and is distinct from the particular form of words in which it is expressed. But this distinction—between proposition and sentence—plays no part in what follows and hence the equivalence here of 'propositional' and 'sentential' logic.

In propositional logic the sentences discussed are restricted to *indicative sentence*. An indicative sentence expresses a proposition about the world, about how things are (or might be in imagination); it indicates things. So, 'All swans are white', and 'All men are mortal', are both indicative. Indicative sentences can be asserted, denied, contended, assumed, supposed, implied, or presupposed. (Indicative sentences contrast with questions and orders, interogative and imperative moods. So 'the door is closed' contrasts with 'Is the door closed?' and 'Close the door!'. Only the first plays a role in propositional logic.)

Thus the basic sentences considered can be true or false.

Logical operators and connectives

The other ingredient of propositional logic is the set of logical connectives or logical operators. A logical operator is a word (or phrase) used either to modify one statement to make a different statement or to join two statements together to form a more complicated statement.. The connectives deployed in propositional logic include 'and', 'or', 'if . . . then'. 'Not' is also an operator, although not intuitively a connective (because it 'connects' to only one sentence at a time).

Some but not all connectives in English are 'truth functional'. These are connectives that, when used to form complex sentences from more basic ones, give sentences whose truth depends only on the truth or falsity of the basic sentences. Thus 'and' and 'or' are (more or less) truth functional in English. (More or less because some uses of 'and' in English imply more than just conjunction but rather something like 'because', which is not truth functional.) The truth of the complex sentences 'A & B' and 'A or B' are given by the following rules:

> A & B is true if and only if A is true and B is true.
> A or B is true if and only if A is true or B is true.

These rules can be symbolized in a 'truth table': the notation devised by the Austrian philosopher, who was based in Cambridge for most of his professional life where he was a professor, Ludwig Wittgenstein (1889–1951). Symbolizing 'and' as '&' and 'or' as '$\vee$' so as to emphasize that these are connectives in the language of propositional logic rather than English and using P and Q to stand for sentences the truth tables are as follows.

P	&	Q
T	T	T
T	F	F
F	F	T
F	F	F

P	$\vee$	Q
T	T	T
T	T	F
F	T	T
F	F	F

The tables set out the truth value of the complex sentences ('P & Q' and 'P $\vee$ Q') for the component truth values of the basic sentences. Because the connectives combine two basic sentences we need to have four rows to capture the permutations of truth

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and falsity (P is T and F when Q is T and P is T and F when Q is F). the truth of the whole sentence: the conjunctive and the disjunctive is given by the column of T and F in the centre.

In the case of '&', the complex is true only if both P and Q are true. For 'or' or '∨' the composite requires that one or the other is true. (Note that 'because', which is not truth functional, could not be expressed in this form. We will come to if ... then shortly.)

The basic connectives are given below. The last pair are if ... then, the material implication, symbolized with the horseshoe and if and only if symbolized ≡.

P	Q	−P	−Q	P ∨ Q	P & Q	P⊃Q	P≡Q
T	T	F	F	T	T	T	T
T	F	F	T	T	F	F	F
F	T	T	F	T	F	T	F
F	F	T	T	F	F	T	T

The basic language of propositional logic is quite powerful. Given the rules for combining basic sentences using connectives, there is no limit to the number of complex sentences that can be formed. There is no limit, for example, simply to conjoining one atomic sentence with others, or even perhaps different tokens of the same sentence type. The resultant complex sentence can then be further conjoined with other atomic or complex sentences. The truth condition of the final result will still be determined by a complex function of the truth conditions of the component atomic sentences and thus derivable from them. But since the process of conjoining sentences can in principle be repeated without limit, a general method of spelling out the truth conditions of arbitrary sentences of the object language will have to be *recursive*. It will have to be repeatedly applicable so that the output of one operation can feed in as the input to the next application until the constituent atomic sentences are reached.

Imagine that a version of propositional logic contains just the following connectives: not, and , or and if ... then. One might then define the 'well formed formulae' of that language by a recursive definition as follows:

1. Any letter standing for a sentence or proposition (e.g. A, B, C ...) is a well-formed formula, wff.
2. If α (standing for A, B , C or for sentences constructed from them using rules 1–6 etc.) is a wff, then so is '$\neg\alpha$' (i.e. not α)
3. If α and β are wff, then so is '(a & b)'
4. If α and β are wff, then so is '$(\alpha \vee \beta)$'
5. If α and β are wff, then so is '$(\alpha \supset \beta)$' (i.e. if α then β)
6. Only sentences according to steps 1–6 is a wff.

A grammar like this, in conjunction with the truth tables for the connectives allows the calculation of the truth value of arbitrarily complex sentences in propositional logic from the truth values of its basic sentences.

Arguing with propositional logic

In fact for many arguments in both philosophy and everyday life the structure of the argument used is quite simple. Thus one tool for assessing whether an argument is valid is by relating it to known valid and invalid forms. Here are some common forms:

Modus ponens

If P then Q
P
Therefore Q

This is a valid argument and perhaps the most common argument form actually used.

Modus tollens

If P then Q
Not Q
Therefore, not P

This is a form of argument used in the hypothetico-deductive model of diagnosis. If the patient has syndrome X then he will have such and such symptoms. He does not have those symptoms. Thus he does not have syndrome X.

Affirming the consequent

If P then Q
Q
Therefore, P

This is a common fallacy. It is not valid. If Socrates is a man then he is mortal. He is mortal. Therefore he is a man. But in fact not all mortals are men and he may be a rabbit.

Denying the antecedent

If P then Q
Not P
Therefore, not Q

This is another fallacy. If patient has syndrome X they will have such and such symptoms. They do not have syndrome X. Therefore they do not have such and such symptoms. But in fact those symptoms may come about for different reasons.

Disjunctive syllogism

Either P or Q
Not P
Therefore, Q

This is again a valid argument.

Reductio ad absurdum

One other form of argument is worth particular note less for its general application than for its more colloquial associations: reductio ad absurdum.

The basic idea is that if by assuming P one can derive an absurdity—such as Q and not Q—then one can conclude that not P is true. This follows from the idea that a valid argument cannot lead from true premisses to a false conclusion. If the conclusion is false—e.g. if it is a contradiction and could not be true—and if the argument is valid, then at least one of the premisses must be false. In practice it is often a matter of debate in real argument, which the false premiss is. (Logic problems generally allow ways to target a particular premiss perhaps by assuming each in turn is false and deriving a reductio for each.)

A very famous reductio is Pythagoras's proof that the square root of 2 is not a rational number:

1. Assume that $\sqrt{2}$ is a rational number, i.e. a number that can be expressed as a fraction *x/y* and, further, the faction is expressed in terms that cannot be further reduced, i.e. divided through by common factors. (The terms in the fraction 4/6 by contrast do have a common factor of 2 and thus that fraction can be reduced to 2/3, which is an irreducible fraction.)
2. If $\sqrt{2} = x/y$ then $(x/y)^2 = 2$
3. Thus $x^2/y^2 = 2$ and $x^2 = 2y^2$
4. Therefore x^2 is an even number
5. Thus x is even (because only even numbers have even numbered squares)
6. If x is even then for some z, $x = 2z$
7. Thus $2y^2 = (2z)^2$
8. Thus $2y^2 = 4z^2$
9. Thus $y^2 = 2z^2$
10. As $2z^2$ is even then y^2 is also even and thus y is even.
11. Thus both x and y are even. So x/y is not irreducible, which contradicts the first premiss.
12. Thus $\sqrt{2}$ is not a rational number.

Assessing validity with truth tables

Truth tables

For more complex arguments, propositional logic has another trick up its sleeve. Return to the key idea that complex sentences can be built using more basic sentences and truth functional connectives according to some rules of grammar. The rules of that grammar allow a complex sentence to be broken up into smaller parts but that still leaves rather a complicated final analysis.

There is, however, an elegant way of calculating the truth or falsity of a complex sentence, which depends on its component parts: truth tables, again. The basic idea is that a truth table sets out the truth value of complex sentences in terms of its constituents. However, the basic truth table can be applied to more complex cases than connections applied to just two atomic sentences. By careful application of truth tables recursively to complex combinations the truth value of the complex sentence can be displayed as a function of its parts. This in turn allows for calculating whether one sentence is implied by another.

The method is quite mechanical. One first calculates the truth values of sentences in the premisses in terms of the basic atomic sentences that make them up, and likewise for the conclusion. Recall that the key test for validity is that the conclusion is never false when the premisses are true. Thus one can see whether the argument is valid by checking that, for a combination of atomic sentence truth values, the premisses are never true and the conclusion false.

Most statements will have some combination of T and F in their truth table columns; they are called contingencies. Some statements will have nothing but T; they are called tautologies. There are true whatever the truth values of their component parts. Others will have nothing but F; they are called contradictions. They cannot be true whatever the truth value of their component parts.

Consider the following arguments:

Not (P & Q)
P
Therefore, not Q

And:

(not P or not Q)
Therefore, not (P or Q)

P	Q	Not (P & Q)	(P & Q)	P	Not Q
T	T	F	T	T	F
T	F	**T**	F	**T**	**T**
F	T	**T**	F	**F**	**F**
F	F	**T**	F	**F**	**T**

The table represents the premisses of the first argument: Not (P & Q) and P in the third and fifth columns of Ts and Fs. The conclusion is the final column. Now the test for validity is that true premisses should never lead to false conclusions. Do they? The premisses are only both true in the second row and then the conclusion is true also. So the premisses are never true and the conclusion false. So the argument is valid.

But the next argument is invalid. The premiss is the fourth column and the conclusion the sixth but in the second and third row the premiss is true when the conclusion is false.

P	Q	(not P	or	not Q)	not	(P or Q)
T	T	F	F	F	F	T
T	F	F	**T**	T	**F**	T
F	T	T	**T**	F	**F**	T
F	F	T	**T**	T	**T**	F

The language of predicate logic

The need for predicate logic

While propositional logic is powerful it cannot capture even everything in the first syllogisms described at the start of this chapter: All men are mortal, Socrates is a man, therefore Socrates is mortal. But there is a more powerful logic devised by Gottlob Frege (1848–1925). We will merely introduce the kind of symbolism used and give directions for further reading on the subject.

Predicates and quantifiers

Frege's innovation was to provide a notation to symbolize a structure within whole basic sentences rather than just relying on whole sentences as building blocks for more complex sentences. This analysis could thus deploy some of the resources of propositional logic (although not the mechanical test of validity based on truth tables) but augment them with further tools.

The basic building blocks of Frege's predicate logic are variable terms standing for objects: x, y, z, etc. terms standing for predicates F, G, H and two quantifiers (universal and existential).

Predicates have one or more places. Thus a one place predicate might be 'Rx' and mean that x is red. Or it might have two places and express a relation between two objects. xLy, or Lx,y might mean x loves y. Note that order may be important. Much sadness turns on the fact that xLy does not imply that yLx!

The quantifiers are the universal quantifier and the existential quantifier. The universal quantifier symbolized ∀(x) says 'For all x' or 'For every x'. Thus ∀(x) Rx says that every x is R, which might mean red: every object in the universe is red.

The existential quantifier ∃(x) says that there is at least one x, or there is some x such that (where some might be singular)… Thus ∃(x) Rx says that there is at least one thing that is R or red, in this case.

Quantifiers predicates and variables can be put together to form whole sentences. Note that every variable in a sentence has to be bound by a quantifier for the sentence to be whole and thus capable of truth or falsity. A sentence that has at least one unbound variable is called an 'open sentence', but it does not actually say anything capable of truth or falsity.

The order of quantifiers

Note the difference between the second and third case below. Simplifying the universe discussed to people the following instances might have the following prose interpretations.

> ∀(x) ∀(y) xLy says: that for everyone x the following is true: x loves everyone y. In other words, everyone loves everyone including themselves.
>
> ∀(x) ∃(y) xLy says that for every person x there is at least one person y such that x loves that person.
>
> ∃(x) ∀(y) xLy says that there is at least one person x who loves every person y.
>
> ∃(x) ∃(y) xLy says that there is at least one person x such that there is at least one person y and x loves y.

The quantifiers thus allow careful articulation of sentences in which in English are ambiguous. Take a textbook favourite example:

> Every nice girl loves a sailor.

This might mean any of:

> There is one sailor that every nice girl loves
>
> Every nice girl loves some sailor or other (not necessarily the same one.
>
> Every nice girl loves every sailor.

The difference between these can be expressed using combinations of the two quantifiers.

We will not, however, go further in setting out the logical inferences that this powerful symbolism allows.

Summary of the session

The purpose of this session has been to introduce some of the key terms and tools of modern logic. Deductive logic seeks valid arguments by careful attention to their form or structure. A valid argument cannot lead from true premisses to a false conclusion. If the premisses are true then so is the conclusion. An argument is sound if its premisses are true and it is valid.

While Aristotle discussed forms of syllogism more modern approaches to logic have been developed from the end of the nineteenth century. Propositional logic is the logic of the connectives and, or, if then, not etc. These are truth functional: the truth values of complex sentences made using them dependent only on the truth values of the components. Truth tables allow the definition of connectives and a mechanical test for the validity of arguments.

Predicate logic refines this structure and adds predicates, variables and quantifies to analyse structure within basic sentences.

In the next session we will examine the nature of logical inference: what underpins it?

Reflection on the session and self-test questions

Write down your own reflections on the materials in this session drawing out any points that are particularly significant for you. Then write brief notes about the following:

1. What is the aim of deductive argument?
2. What is meant by 'valid' and 'sound' as applied to arguments?
3. How can the validity of arguments be assessed?
4. What forms of deductive argument or logic are there?
5. What is propositional logic? And what is a truth table?

Session 2 An introduction to the philosophy of logic: what underpins deductive logic?

Deduction and induction

Thus far, we have been considering a form of inference known as *deduction*. The codification of deductive validity in propositional logic suggests that its strength lies in an essential modesty: the conclusions of deductive are somehow contained within the premisses. The arguments simply unpack something that was already implicit in the premisses. It is this very fact that ensures that true premisses cannot lead to false conclusions. By comparison

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with another kind of argument, however, that can seem very modest indeed. We will argue briefly, however, that deduction still has a clear use. We will then consider more deeply what underpins logical inference, and what precise sense there is in saying that the premisses contain the conclusions.

The other central form of inference is *induction*. Where deductive reasoning relies only on the definitions of the terms in which it is conducted, inductive reasoning relies on the way things are in the world. An oft quoted example is this: 'because the sun has risen in the East for millions of years, therefore it will rise in the East tomorrow'. Inductive reasoning, being in this way experience based, is crucially important to scientific reasoning. In induction, the premisses supply only a part of what appears in the conclusion. That is to say, the induction is *ampliative*, in that it goes further than deductive argument will allow.

Deduction is uninformative, circular, and tautologous

All deductive inference has three major features that distinguish it from inductive inference. These three features are that it is, strictly, *uninformative, circular*, and *tautologous*.

Deductive inference is *uninformative* in a quite straightforward way: in not allowing us to go beyond the information contained in the premisses, it does not offer us any new information about the world.

A *circular* argument argues in a circle. It ends where it started from. But circularity is not necessarily a bad thing. Deductive inference is such a powerful tool of reasoning just because is allows us to check the correctness of a claim, given certain starting assumptions. If it were not the case that this involved a certain amount of circularity (as in syllogism (2) above), then a valid deduction would not be compulsive. Valid deduction is compelling in that we cannot deny that the truth of the conclusion is a logical consequence of the truth of its premisses, in virtue of its form. But it does suggest a constraint on how deduction can play a useful role. Consider:

> All swans are white
> Cygnus is a swan
> Cygnus is white.

Suppose that knowing the truth of the first premiss involved or required checking each swan. Then it seems that the only way to know the truth of the premiss would be to check each and every swan *including Cygnus*. And that seems to make the argument useless. There is no use for the argument given that knowledge of the premiss requires prior knowledge of the conclusion. But, as we will see, there are still uses for such argument.

Tautology

We suggested that the third characteristic of deductive argument was that it is tautological. This can seem surprising. A tautology, on a standard dictionary definition, means something like: saying the same thing twice over. That also does not sound very useful but as we will see it does not undermine the purpose of deductive argument.

In logic a tautology is a defined as a complex statement whose truth is independent of the truth or falsity of its component parts. As a result it does not tell us anything about the world. Consider the statement:

> Either it is Monday or it is not Monday.

That whole sentence is true if either of its parts is true. These are:

> It is Monday
> It is not Monday

Now if it is Monday, the first component is true and so is the whole sentence. But in all other cases, the second component is true and so, again, the whole sentence is true. Thus its truth is independent of the truth or falsity of the components. Furthermore, the fact that it is true tells us nothing about the day of the week.

Valid arguments are, however, related to tautologies in this way. Consider the argument,

> P
> If P then Q
> Therefore, Q

This is valid. This means that if the premisses are true then the conclusion is true (it must be true). This means that a single whole sentence that conjoins the premisses and connects them to the conclusion with the connective if . . . then will always be true. So the sentence

> 'If (P and (if P then Q)) then Q'

is always true whatever truth values of P and Q.

Consider now the argument:

> It is Monday, and
> If it is Monday then the weekend is at least four full days away
> Then the weekend is at least four full days away.

This is a valid argument. If the premisses are true then so must the conclusion be. But the argument is clearly related to the following statement:

> If, it is Monday and, if it is Monday then the weekend is at least four full days away, then the weekend is at least four full days away.

This statement is tautological. The structure of the sentence is so formed that whatever day of the week it is, the sentence as a whole is true.

Whenever we can frame a valid argument we can translate it into a tautological statement. It is the validity of the argument that guarantees this.

The role of deductive argument given that it is 'uninformative'

If the conclusions of deductive arguments are strictly uninformative and circular what is the purpose of deductive inference? In scientific inquiry there are at least three positive uses (which in turn motivate examination of logical inference and argument).

1. Scientific theories and hypothesis make *general* claims. Testing such theories, however, relies on the examination of (some of) their consequences in *particular* circumstances. As the general claims cannot be directly tested as a whole it is necessary to *derive* particular consequences for experiment and observation.
2. Deep explanatory theories often postulate unobservable or hidden causes. (These may unify diverse observable phenomena.) As the claims about unobservable causes cannot be directly tested (by definition), testable claims need to be derived from them. Thus there is a positive advantage in the fact that the deductive move from general theory to particular claim involves a loss of information (about the hidden cause).
3. Claims about the world couched in ordinary sentences deploy concepts that have implicitly general consequences. To say that there is a particular chair in a particular corner is to make a particular claim. But it also implies—because of the implicit generality of the concept 'chair'—that there is something substantial, enduring, of mass, etc. in the corner. Drawing deductive conclusions is one way of highlighting the aspects of that content with which we are interested in a particular context.

Consider just the first of these cases. Diagnosis in general practice is often described as following a hypothetico-deductive pattern. Presented with the signs and symptoms, one forms a provisional list of possible diagnoses. Starting with the most likely, one derives an observable deductive consequence. If the symptoms result from such and such underlying cause then that cause would also produce so and so. Is so and so present? Yes or no? This process relies on deduction in a way that escapes the worry about circularity mentioned above.

Can one ever reach a conclusion?

We have so far briefly outlined three ways to codify logical argument: syllogism, propositional and predicate logic. And we have flagged the apparent weakness of deductive argument: that it simply unpacks its premisses. But that way of justifying deductive argument faces the following historically significant objection.

(10 minutes)

Look back at the reading with Exercise 2 from *What The Tortoise Said To Achilles* by Lewis Carroll and the continuation that goes with this exercise.

Link with Reading 5.1

How does the Tortoise generate a sceptical challenge against the rational force of logic? What is the significance of his comment: 'Whatever *Logic* is good enough to tell me is worth *writing down*,'? How could the scepticism be defused?

Summary of the problem

The bare bones of the Tortoise's challenge can be thought of like this. The Tortoise asks what the conditions are for being rationally convinced by the argument 'A, B therefore *Z*' of the truth of the conclusion *Z*? He suggests that there are two components.

1. Rational belief in the truth of the premisses and . . .
2. Rational belief in the validity of the sequent.

The Tortoise accepts (1) but asks to be persuaded of (2). It seems reasonable to suppose that if logic is to be a means of persuasion then there should be the resources to convince him.

But, in fact, the Tortoise suggests that there is a regress. The problem is that rationally believing (2) amounts to accepting that:

C: If *A*,*B* then *Z*

So the Tortoise must necessarily accept *C* as well as *A*,*B* if he is to be rationally persuaded of *Z*. Because what logic is good enough to teach is worth writing down he adds this to the list of claims he has to believe alongside the first two premisses. Strangely while a moment before he did not believe C he is willing to grant it when asked by Achilles, proving it is written down. It might seem—as it does to Achilles—that this has now solved the problem.

But now the argument seems to have changed to:

A, *B*, *C* therefore *Z*

and the Tortoise now says that rationally to believe *Z* he must necessarily accept the new hypothetical:

D: If *A*, *B*, *C* then *Z*

Now there is a regress. It appears one has to accept an infinite number of premisses before one can be rationally persuaded of *Z*. Thus one cannot be rationally persuaded. And thus in general, logic cannot rationally persuade!

It is worth pausing at this point. If the Tortoise is correct then logical inference is like Zeno's paradox, as he mentions at the start, but without the solution that the steps involved in getting to the end (the end of the race or the conclusion of the argument) get smaller each time and can be summed. It seems that Carroll wrote this piece because he was genuinely confused.

Diagnosis

In general in philosophy if a sceptical argument is deployed it is wise, if possible, to try to block it before it gets off the ground. In this case the addition of premiss *C* that codifies the hypothetical involved is what starts the problem off.

Note first that in general, adding extra premisses that codify the hypothetical involved to arguments is not a good idea. Consider two cases:

- If the argument is valid already then adding the extra hypothetical premiss is unnecessary. If it was valid before then it will be no more valid afterwards.

- If the argument was invalid before then adding the premiss will make it valid. But recall that there are two ingredients for rational persuasion: belief that the argument is valid and belief that the premisses are true. The strengthening extra premiss in this case reports that the prior argument is valid. But that is false. So one of the premisses of the new argument is false. The argument is not sound. And thus *Z* does not follow.

The moral that Carroll should have drawn is this: the Tortoise confuses grasp of rules of inference with acceptance of further premisses. They are distinct. Thus although the Tortoise must accept the rule equivalent to the hypothetical *C*, this is not a further premiss.

Justification of an argument is a meta-level commentary on that argument, not an extra step in the same argument. The correct way to persuade the Tortoise is not to ask him to accept a further premiss but to show why the argument is valid in the first place, it is to show that it has the correct form.

But a problem remains

The purpose of looking at Carroll's brief article is not, ultimately, to call the force of logic into question but to shed light on how it has that force. It is not a matter of implicitly knowing an infinite number of premisses. Nor is it a matter of external compulsion.

Recall Achilles rather desperate cry: 'Then Logic would take you by the throat, and *force* you to do it!' That is a natural expression of a platonist view. Logic can be pictured as wholly independent of human subjects. But Achilles's hope is in vain. (We will return to Wittgenstein's criticism of platonism in Chapter 25.) Rather, if one has eyes to see it, then one can see that the premisses are indeed sufficient for the truth of the conclusion without the need to pile up further assurance.

That, however, still leaves a question: What is it that underpins the transition from premiss to conclusion? We can ask this not in a sceptical tone but for clarification of the nature and origins of logical force.

In what sense is the conclusion of an argument contained in its premisses?

(10 minutes)

Look at the extract from the following reading:

Prior, A. (1960). The runabout inference-ticket. Analysis 21: 38–39 (Reprinted in Strawson (ed.), *Philosophical Logic*. pp. 217–218)

Link with Reading 5.3

- What is the analytic theory of validity, and how does the connective tonk undermine that picture?

The analytic theory of validity

Prior's target is a family of views about the nature and underpinnings of logic that he calls the 'analytic theory of validity'.

This is the intuitive and attractive view that logical inferences are sustained solely by the definitions of the logical connectives involved. Connectives and inferences are completely mutually defined *de novo*. The inferences fix the meaning of the connectives and the connectives fix the inferences allowed. Neither answer to anything else. Logic is a kind of self-contained game.

Imagine that someone were to ask why the knight in chess can move in the way it can? Now in a modern design of chess set it might be reasonable to ask why 'that piece!' has moved as it has and the correct answer might be that looks notwithstanding it is a knight and thus that is a move it is allowed. But identification of the chess piece is not an issue then there is no very helpful answer to give except to say that knight's are just defined in the game of chess by being permitted that move. That is what makes it a knight and because it is a knight it can move in that way. Moves and pieces are defined together and answer to know uber-chess facts. Chess is not, for example, a representation of a real battle. It does not have to describe anything.

So the analytic theory gives an austere answer to the question of what sustains logical inference: the mutual definition of connectives and inferences.

Tonk's challenge

Tonk is deployed in a kind of *reductio ad absurdum*. Its definition fits the analytic theory of validity. However, it leads to the absurd conclusion that anything may be inferred from anything else. Thus it is absurd and thus so is the analytic theory.

> 'Tonk' is defined thus:
> A implies A tonk B, and A tonk B implies B.
> Thus A implies B for arbitrary A, B.

But from the perspective of the theory, there should be nothing wrong with these rules of inference given the meaning of tonk and the meaning, given the rules. There is no other standard of correctness.

What response should we give to this challenge? We will consider two.

Belnap's context of deducibility response

In a brief paper in the philosophy journal *Analysis*, which specializes in very short pieces, the logician Nuel Belnap (1961–62) offered the following diagnosis. First, he suggested that there could be no such connective as tonk just as there could be no such mathematical operator '?' defined as follows. For any fractions a/b and c/d then,

$$\{a/b \; ? \; c/d\} = \{(a + c)/(b + d)\}$$

The problem with the function '?' so defined is that it gives different results, e.g. {2/3 ? 4/5} and {4/6 ? 4/5}. (Calculate the result for both!) But 2/3 equals 4/6 and thus it should not matter which one picks.

Belnap says that this definition contradicts prior assumptions about mathematics (that e.g. 2/3 = 4/6). From this he draws the moral that new mathematical functions have to be defined in a way that is consistent with—or conservative of—prior assumptions.

Likewise, in logic, new connectives should be defined in terms of inferences in such a way that inferences not involving them do not permit inferences that were not possible before. They should preserve the context of deducibility.

The rules governing 'tonk' fail because they allow the inference of statements *not involving tonk*, which were not previously deducible by the rules of deduction. The reason for saying statements *not involving tonk* is this. The inference of the 'contonktive' A tonk B from A does not violate prior assumptions. This was not a statement that had any meaning before tonk was introduced. It is the next stage: deriving B from the contonktive, that was not possible before but which violates logical inference.

This raises the general question, however: What characterizes the context of deducibility? Belnap suggests that all new rules must be conservative extensions of a context of deducibility defined: Gentzen's structural rules.

Gerhard Gentzen

Gerhard Gentzen (1909–45) worked on logic and the foundations of mathematics. His structural rules are very general rules that are supposed to characterize permitted rules of inference. They are fairly intuitive allowing, e.g. the addition of extra premisses to already valid arguments (cf. the Tortoise above) and swapping round of premisses within arguments. These are the rules:

weakening: from A1,…, An ⇒ C infer A1,…, An, B ⇒ C

permutation: from A1,…, Ai, Ai +1 ,…, An ⇒ B infer A1,…, Ai + 1, Ai,…, An ⇒ B

contraction: from A1,…, An, An ⇒ B infer A1,…, An, ⇒ B

transitivity: from A1,…, Am ⇒ B and C1,…, Cn, B ⇒ D infer A1,…, Am, C1,…, Cn, B ⇒ D

It is outside the scope of this chapter to discuss whether they are successful in characterizing the 'context of deducibility'. But in the context of this chapter—the context of asking what underpins logical inference—they do raise the following problem. They simply repeat, albeit in a more abstract structural way, a list of permitted inferences. And that leaves open the question: What underpins these rules? (Again the question is not being asked in the Tortoises sceptical way. Rather, the question is what in general underpins inferences?) Simply listing more rules does seem to address this question.

Stevenson's vindication through truth tables

In another short paper in the journal *Analysis*, Stevenson (1961) provides a different response to Prior (1960).

He points out that tonk is defined as a kind of permissive game, without any consideration of the injunction that should characterize logic: that truth be preserved in a valid argument. True premisses should never lead to a false conclusion. That is the essence of logical validity set out in the previous session. Stevenson suggests that if an inference is called into question—as, e.g. the Tortoise does—it should be *validated* by showing that it follows a rule and then that rule should be *vindicated*.

How should a rule be vindicated? Stevenson argues that this can be done with a truth table. One can spot that something is wrong with tonk because no single truth table can represent how it functions. The introduction of tonk (from A infer A tonk B) looks like 'or' and the elimination (from A tonk B infer B) looks like 'and'. Thus it needs two truth tables.

So a condition of possibility for connective introduction is that they can be represented by a single truth table. This is a sensible response to tonk. But it leaves open the question of whether Stevenson's response undermines the analytic theory of validity—i.e. agrees with Prior—or whether it merely augments it. In other words can we say that inferences and connectives are mutually defined, answer to nothing else, but that the definition has to meet a particular internal consistency constraint: they must be codifiable in a truth table? There is no clear answer to that question.

(Comment on vindication by truth table)

Although Stevenson (1961) provides a sensible criticism of tonk, and it can serve as a necessary test of a potential connective, there remains a general problem with using truth tables to justify rules of inference. This was made clear in a paper by the philosopher Sue Haack (1976). She suggests that typically, at least, the application of the truth table will use (at the meta-level) the very (ground-level) rule to be justified. So as a general method of vindication truth tables only work if one can already presuppose correct rules of inference and that is question-begging.

She gives the following example using Modus Monens and contrasting it with the fallacy of affirming the consequent (symbolized MM in her paper):

A1: The justification of Modus Ponens (using the truth table for if..then.)

Suppose 'A' is true and 'A ⊃ B' is true.

By the truth table for '⊃' if 'A' is true and 'A ⊃ B' is true, then 'B' is true too.

So 'B' must be true too.

This argument has the form:

A1*
Suppose C (that 'A' is true and that 'A ⊃ B' is true).
If C then D (if 'A' is true and 'A ⊃ B' is true, then 'B' is true).
So D ('B' must be true too).

Now Haack (1976) suggests that this justification is akin to inductive justifications of induction (which will be discussed in Chapter 16).

A2: Justification of induction:
Induction has usually been successful in the past.
Therefore (by induction) induction is usually successful.

This inference assumes the very rule of inference—induction—it is designed to justify. Is this circularity vicious or virtuous? It is hard to tell. But the problem is that counter-induction is also self-justifying. Counter-induction is the principle that what has happened in the past will not happen in the future. It can also justify itself:

A3 Justification of counter-induction:

Counter-induction has usually not been successful in the past.

Therefore (by counter-induction) counter-induction is usually successful.

So if there is a problem with induction—and many philosophers think there is as we will discuss in chapter 16—Haack suggests that there is in deduction as well. Because justifying Modus Ponens—perhaps the key form of inference—using truth tables invokes that very principle. But is there an analogue of counter-induction for deduction? Yes: affirming the consequent (MM in what follows).

Deductive analogue of counter-induction:

Define MM: from A ⊃ B and B infer A

A4 Suppose that 'A ⊃ B' is true and 'B' is true, then 'A ⊃ B' is true ⊃ 'B' is true.

Now by the truth table for '⊃', if 'A' is true, then if 'A ⊃ B' is true, 'B' is true.

Therefore 'A' is true.

To make this a little clearer it has this form:

*A4** Suppose D (if 'A ⊃ B' is true, then 'B' is true)
If C, then D (if 'A' is true, then, if 'A ⊃ B' is true, 'B' is true).
So C ('A' is true).

But we will not consider this worry further here.

Analyticity

So something like the analytic theory of validity remains attractive. It is prompted by a response to the Tortoise that you may have arrived at yourself in thinking about his problem. Recall, the Tortoise questions why he should believe the conclusion *Z* given that he accepts the premisses A and B. Recall also what the argument actually was:

(*A*) Things that are equal to the same are equal to each other.
(*B*) The two sides of the Triangle are things that are equal to the same.
(*Z*) The two sides of this Triangle are equal to each other.

Now one response is to say that if the Tortoise accepts *A* and *B* then he has *already* accepted *Z*. *Z* is already contained with *A* and *B*! But what is the sense of 'containment'? Premisses are not literally containers.

We will consider this response a little more.

Analytic versus synthetic

The terms *analytic* and *synthetic* were introduced by the eighteenth century Prussian philosopher Immanuel Kant. A statement expresses an *analytic truth*, only if the concept of the predicate is contained within the concept of the subject. For example, 'All husbands are male' is composed of a subject term ('All husbands') and a predicate term ('male'), with a connective ('are') that functions as the logical operator of predication (i.e. 'have the property of 'x'-ness' where 'x' = 'male'). But because the idea of maleness is contained in the idea 'husband', an analysis of the latter necessarily reveals the idea of maleness.

Thus an analytic truth is a truth that depends upon meaning alone. By contrast a synthetic truth depends on meaning (to be the truth it is) but also on how the world is. Thus 'all bachelors are happy' is true in virtue both of what the terms mean but also, given what they do mean, on whether bachelors are, as a matter of fact, happy.

Necessary versus synthetic

There are two other influential distinctions between kinds of truth. The analytic synthetic distinction is a distinction within semantics. It has to do with the relation of truth and meaning. The other two are metaphysical and epistemological.

A truth is necessary if it could not have been false and contingent if it could have been false. A necessary truth is true in all possible worlds. A contingent truth is true only in some possible worlds, worlds most like ours.

A priori versus a posteriori

Arguably, some truths can be known only through experience. Other truths can be known without experience (whether or not they could also be known through experience). This is the epistemological distinction between the a priori—which is knowable prior to experience—and the a posteriori—which can be known only after experience.

(10 minutes)

Consider the following thought experiment. Suppose that you had access to all the truths about the world, the earth, its place in the cosmos, the truths of reason, etc. Imagine that you were to divide the truths up using the three distinctions above: semantic, metaphysical, and epistemological. Would they divide the truths in the same way?

Why might they align?

Many philosophers have thought that the three kinds of truth will divide truths up in the same way: philosophers from Hume—in so far as we can ascribe a semantic thesis to him—to the Logical Positivists of the early twentieth century. Here is an argument why they might:

1. If we can know a truth a priori we can know it without knowing which possible world we inhabit.

2. Thus is must apply in all possible worlds.
3. Further, it must be something we bring to knowledge of the world rather than being given by the world.
4. So it must be analytic.

To find truths which do not fit this alignment we need to think of, e.g. synthetic but a priori truths or a posteriori but necessary truths. And indeed two philosophers have made such claims.

Kant argued that the truths of arithmetic were a prior. But they were not analytic. No amount of thinking about the concept of 7 and plus and 5, for example takes one beyond 7 and plus and 5. It takes a further intuition to get to 12.

More recently, Saul Kripke (1940–) metaphysician and logician has argued that there are a posteriori necessities. If water is H_2O then it is H_2O necessarily. There is no possible world in which there is water but no H_2O. Of course there may be possible worlds in which there is something that looks and tastes like water; however, if it is not H_2O it is not water. Still, it took much empirical work to discover that.

In Part V we will look at Kripke's argument that mental states are not physical states for reasons based on this claim.

Quine's attack

W.V.O. Quine has argued, however, that not only do the distinctions between truths not align, there are no such distinctions. He argued this in a seminal paper: 'Two dogmas of empiricism', which often tops the polls of most influential philosophical paper among analytic philosophers.

The kernal of Quine's argument is that we cannot explain what we mean by analyticity. He claims that there is no distinction between the analytic and synthetic on the grounds that we cannot explain what that distinction amounts to. All attempts at explanation presuppose equally mysterious concepts.

Thus for example, he argues that we cannot define analytic truth by appeal to logical truths. We might have thought that 'all bachelors are unmarried' might be derived from the logical truth 'all unmarried men are unmarried' via substitution of the synonym 'bachelor' for' unmarried man'. But, Quine argues, synonymy is just as dubious as analyticity.

We cannot attempt to explain that term through the notion of definition because most, at least, definitions attempt to capture prior relations of synonymy rather than explaining that notion.

We might have attempted to define synonymy by noting that when one substitutes a synonym for a word in a sentence then its truth or falsity is unaffected. (Of course this is not true if the sentence, e.g. refers to the number of letters in a word.) So one might have assumed one could turn this around and define synonymy as that which leaves the truth of sentences unchanged when substitutions are made. But in fact it needs to be stronger than that. Truths about 'creatures with a heart' will be the same as truths about 'creatures with kidneys' because they are the same creatures, as a matter of fact. But those phrases do not mean the same. But any strengthening of the relation will require using terms such as 'of necessity' ('the substitution is truth value preserving of necessity'), which Quine asserts, are just as obscure.

Quine concludes that because analyticity can only be explained in terms of a small number of equally opaque concepts, there is no such concept. Why does this follow? Quine does not himself say but one thought is that if there is no way of testing correct understanding, there is nothing determinate to be understood.

Quine's conclusion is radical but influential. There are no analytic truths. What look like analytic truths are simply truths at the centre of our web of beliefs or concepts. We might revise them but to do so would require revising many other concepts and beliefs. Logic too is simply a central part of the structure of our thought but, like any other part of that structure, could be revised in the face of experience.

Analyticity and containment

The problem Quine raises in answering the question of this session though is this: What sense can we give to the idea that logical validity is explained by the idea that the conclusion to a valid argument is contained within the premisses? Usually it is not explicitly contained there. So the notion of containment is a kind of metaphor. And analyticity might have 'unpacked' that metaphor. But Quine threatens that idea.

Note also that even in arguments where the symbols used in the conclusions are also in the premisses this will not help. Consider the argument:

Not P
Therefore, P

Here 'P' is in both but that does not make it a valid argument.

In fact whether Quine's argument is successful or not is very much a matter of debate. The Oxford philosophers of language H.P. Grice and P.F. Strawson raised a number of objections in a seminal paper (see the further reading). A key point they make is that Quine puts two restrictions on explanations of analyticity:

1. They should not use related expressions.
2. The explanation should be of the same general character as the related expression.

In a piece of English understatement they suggest that such explanations will be 'hard to come by'. What they mean is that the requirements are incompatible and thus, with them in place, it is no surprise that no explanations of analyticity can be given.

More recently, the American philosopher Hilary Putnam (2002) has argued that a distinction can be drawn between distinctions and dichotomies. A dichotomy requires that cases it concerns all fall on one side or another. A distinction, by contrast, does not and can simply be a way of dividing cases for a particular purpose, in a particular context. Putnam argues that there can be a distinction between the analytic and synthetic even if not a dichotomy. (This is also discussed in Chapter 2.)

Thus it remains very much a matter of debate whether the notion of analyticity can really be used to explain the intuitive idea that the premisses contain the conclusion. That notion of containment may be best explained by pointing to valid logical inferences. But if so it cannot itself explain the force of logic.

Reflection on the session and self-test questions

Write down your own reflections on the materials in this session drawing out any points that are particularly significant for you. Then write brief notes about the following:

1. In what sense is logic uninformative?
2. Does this undermine deduction?
3. What underpins logical inference?
4. Can truth tables be used to justify inferences?

Session 3 Conclusions: implication and entailment

Formal and informal logic

This chapter has concerned *formal logic*.

Formal logic is concerned with the *form* of arguments, and the focus is upon the abstract structure of the most general kinds of claim. This helps avoid distraction by the *content* of the statements under consideration. The aim is to arrive at a complete, consistent and transparent axiomatic system of logical relations and principles (i.e. a system derived from a few axioms, which, rather as in classical geometry, are an agreed starting-point—one of the problems in logic has always been to agree on which axioms are in this sense basic!). Formal logic is *context-free* in that what makes an argument valid is independent of any specific context, and it is usually presented in the form of symbolic notation (more on this shortly).

Informal logic, on the other hand, eschews recourse to symbolic notation, and does not aim at the elucidation of a perfect logical system, or the derivation of rules of inference from a set of axioms. Its ambitions are modest in that it seeks only to clarify and render more precise the argumentative strategies recognized and accepted as valid in ordinary usage. It is therefore diagnostic in its approach, and deals exclusively with examples of everyday language. As a result, informal logic cannot remove itself from concern with the *content* of the arguments it studies, as an important aspect of its inquiry is into the way in which the *context* (i.e. the setting of the content) of an argument can influence the validity of application of a logical construction or form.

Equivocation

Consider the following hypothetical syllogism:
If I had a high forehead I would be intelligent
If I were an elephant I'd have a high forehead
Therefore, if I were an elephant I'd be intelligent

Now, as it stands, this argument appears valid. Translated in formal logic it looks valid, whether or not its premisses are true (so it may not be sound). But informal logic, on the other hand, is concerned with the context of an argument, and, as this example suggests the context of an argument can have an important bearing. Informally we have reason to doubt this argument.

The problem here is one of *equivocation*: it is unclear that the possession of a high forehead in elephants means the same thing as the possession of a high forehead in humans—so even if it were true that being a human and having a high forehead was correlated with intelligence, it is not necessarily true that being an *elephant* and having a high forehead is correlated with intelligence. There is thus equivocation in the use of the relation 'high forehead and intelligence' between the human context and the elephant context. We cannot assume that the relation is the same in both contexts, and that therefore the meaning is the same. And if we cannot assume this, the argument doesn't work—it has committed the *fallacy of equivocation*.

Thus one of the aims of formal logic is to escape this danger. But, as we will see, there is a cost to formal codification.

Ordinary implication and everyday fallacies

Logic is concerned with what follows from what. To argue to a conclusion is to presuppose that there is a relation between the starting-point of the argument and where it ends. This relation, of something (logically) following from something else, is commonly known as *implication*. For example, in syllogism (2), the two premisses seem to imply the conclusion:

2. All men are mortal
   Socrates is a man
   Therefore Socrates is mortal

This seemingly straightforward idea (of one thing following logically from another) has been subject to intense scholarly debate concerning its exact properties. The problem has been to try to formalize the relation in logical notation. So far no totally satisfactory account has been produced. But because of the importance of the mechanism of implication for *any* argument, some knowledge of its properties is essential. While such knowledge will not guarantee immunity from error (all humans are prone to faults of reasoning), it will enable one to avoid the more egregious errors, i.e. to be able to recognize the obvious fallacies is to be forewarned!.

At the end of this chapter we give a list of the more common forms of invalid or fallacious argument (see Appendix). We also suggest an exercise that will get you into the spirit of fallacy-spotting. Many of the fallacies cited are well-known, and even have names. It is remarkable, though, how commonly fallacious arguments are used, and to very good (or bad!) effect. So this is one area in which philosophy, by helping to sharpen up our thinking skills, can be useful in practice.

Problems with codifying ordinary informal implication

One problem for codifiying implication stems from the way it is used in everyday reasoning. Consider the statement: 'If I water this seed, then it will germinate'. This is in the form 'If *A* then *B*'. It is an example of an ordinary informal implication, and it does indeed sound as if it might be true.

But note that the inference 'I have watered this seed therefore it will germinate' is not formally valid, because there is no *logical connection* between the statement 'I have watered this seed' and 'therefore it will germinate', which will ensure that germination will indeed follow the watering of the seed. The relation between the watering of a seed and its germination is determined by the properties of the water and the seed, not anything to do with logical form. That seeds germinate following watering is, for example, only *contingently* true and not true if the seeds are also subject to very high levels or heat or radiation in addition to being watered.

Material implication

We have already seen that formal logic in the shape of propositional logic concentrates on truth-functional connectives. This is the case with the account given in the first session of 'if . . . then' (symbolized by the horseshoe) and which represents 'material implication'

The idea behind this is to try to capture what is meant by 'if . . . then' in English but in truth-functional terms. And this is what causes problems.

The codification gives conditional argument the minimum possible logical force, so that '$p \supset q$' simply asserts only that it is not in fact the case that *p* is true and *q* is false. That is what the truth table codifies. It can be summarized as equivalent to 'Not (*p* & not-*q*)', or ~(*p* & ~*q*).

Unfortunately, problems remain. They are known as the *paradoxes of material implication*, and their origin lies in the role of '−' and '⊃' as *truth-functional operators*.

Truth-functional operators

Recall the truth table for 'if . . . then'(⊃):

p	$\supset$	q
T	T	T
T	F	F
F	T	T
F	T	F

This truth-table defines the relationship between '*p*', '*q*', and '⊃'. It shows that the material conditional may be true even when there is no connection whatsoever between its antecedent and consequent, e.g. 'If elephants are pink, then the sky is blue' (i.e. the third line down: FTT). In other words, a true conclusion can follow from a false premiss, under the rules of material implication.

It thus permits true conclusions to be inferred from irrelevant premisses but it also allows us to draw inferences about non-existent objects.

More generally, for any statement *A*, if *A* is false, '$A \supset B$' is true; and if *A* is true, '$B \supset A$' is true, no matter what statement *B* is. The paradoxes emerge because it is possible to assert that a conditional that contains a false propositions as either a premiss or as its conclusion, or both, is true. So all the following would be true:

> If elephants are pink, then the sky is blue
> If elephants are pink, then the sky is not blue
> If elephants are not pink, then the sky is not blue

The problem is that capturing the notion of implication in merely truth-functional terms does not capture the relevance of the premiss to the conclusion.

Strict implication

We can sum up the problem thus: as a simple matter of *fact*, it may be true that 'If *A* then *B*' (i.e. it may be true according to its truth table definition), but it does not follow that the inference '*A* therefore *B*' is valid in a more intuitive sense. It may not be true that *B* follows from *A*. Everyday entailment seems not to be a matter merely of the truth and falsity of sentences in premisses and conclusions, whereas materal implication is truth functional.

One intuition is to say that, for the inference from *A* to *B* to be valid, is for it to be *impossible* for *B* to be false when *A* is is true. One way of achieving this is to define the relation in terms of *necessary* truth. This is called *strict implication*.

The Oxford philosopher, C.I. Lewis, writing in the 1950s, introduced the notion of strict implication in his system of *modal logic*. Modal logic affords a distinction between ordinary truth and *logically necessary truth*. In strict implication, 'P strictly implies Q' means 'It is logically necessary that P materially implies Q'. This is the reasoning behind it. Consider:

1. If elephants are pink then the sky is not blue.

Clearly (1) is false, because the colour of elephants has no bearing on the sky. Now one can represent (1) as

2. elephants are pink ⊃ the sky is not blue.

But (2) is true because the antecedent is false and thus it is not a good translation of (1). Lewis's is suggested translation is:

3. [] (elephants are pink ⊃ the sky is not blue.)

This says (roughly) that in every possible world where elephants are pink, the sky is not blue. As one can easily imagine a world of pink elephants and a blue sky, (3) is false. Hence, it seems a better of (1).

This avoids the paradoxes of material implication, but entails similar paradoxes of strict implication. Namely, an impossible statement P strictly implies any statement Q, and a necessary statement Q is strictly implied by any statement P. Consider:

4. If elephants are pink then $2 + 2 = 4$.

Rendered formally with strict implication, (4) becomes

5. [] (if elephants are pink ⊃ 2 + 2 = 4)

which says (roughly) that in every possible world where elephants are pink, 2 + 2 = 4. Because it's impossible for there to be a world where 2 + 2 fails to equal 4, (5) is true. But surely (4) is false.

Nuel Belnap (1961) offered a very strict and conservative relation analysing premisses and conclusion into Boolean 'atoms' and requiring the literal containment of the conclusion-atoms among the premiss-atoms. But their relation eliminates all the paradoxes mentioned so far, but ruthlessly also eliminates much else that seems innocent, in particular, the rule of inference called disjunctive syllogism: P or Q, not-P, therefore Q.

Implication and entailment

The continuing debate about how best to analyse implication in logical terms presents some of the strengths and weaknesses of modern deductive logic.

On the one had it presents a series of powerful tools—which this short introduction can merely point towards—for establishing rigorous argument and for disambiguating ambiguous claims made in everyday English. By articulating forms of argument it can provide the basis of a critique of reasoning that is impartial and independent of particular contents. One cautionary aspect of this is that by looking to the structure of argument one can also highlight fallacies: forms of argument that do not preserve truth. The Appendix lists some of these and learning to identify fallacies as well as valid arguments is a valuable thinking skill.

But on the other hand there is a cost to translating our everyday terms and ideas into the rigorous but minimal language of formal logic. It is particularly striking that one of the most difficult translations is of the central aim of logical argument: implication!

Back to the opening quote

We began this chapter with Achilles's cry that Logic would compel the Tortoise bodily: it would take him by the throat. In fact, while the Tortoise's particular challenge seems to have been based on a misunderstanding, the actual force of logic is more mysterious than we might have thought. But some aspects are very clear. Logical validity is based on the form or structure of arguments and if an argument is valid it cannot lead from truth to falsity. That guarantee comes at a cost: the conclusion to an argument does not contain more information than the premisses. A statement of the argument is a tautology: it cannot but be true but that is because it is independent of the world. Nevertheless logical inference plays a vital role in teasing out implicit assumptions or testing hypotheses. But at the same time, if the conclusions of arguments are contained in the premisses that is so only for subjects with eyes to see it, subjects able to play the game of logical reasoning. Acquiring that vision is a key thinking skill developed through practice in argument and exercises. The Appendix below outlines some logical fallacies, recognition of which is one aspect of that skill.

Reflection on the session and self-test questions

Write down your own reflections on the materials in this session drawing out any points that are particularly significant for you. Then write brief notes about the following:

1. What is the key strength of formal logic? What is a key limitation?
2. What central paradox is raised by the logical codification of implication and of the connective 'if . . . then'?
3. What possible solutions have been proposed?

Appendix Some of the more common forms of invalid argument

In the 2500 years during which logic has been studied, a variety of fallacies and invalid forms of argument have been identified. Some of the more common ones are given below.

Spotting fallacies or invalid arguments is a skill like any other, and as with all skills, it is practice that leads to competence. So it is worth getting into the habit of looking for logical errors! The list here is necessarily in a very summary form. There is no single source for reading more about these. Many philosophical dictionaries include at least some of them and a good general account is Walton's (1989) *Informal Logic*, given in the suggestions for Further reading at the end of the chapter. However, there is no need to memorize the list. It is helpful to have an idea what philosophers are talking about when they use the names given below. But the important thing is to develop a sharp eye for misleading or otherwise fallacious arguments (in your own work as well as other people's!)

(30 minutes)

Take any everyday piece of text (this could be a newspaper, a brochure, etc.) or record a discussion programme on the TV/radio:

1. note down any argument you see/hear which seems to you fallacious; then,
2. go through the following list and see if the fallacies can be identified there; finally,
3. return to your text/recording, and see if you can spot any further fallacious arguments.

Ad baculum

The rhetorician's tactic of arousing strong emotions in the audience as a persuasive device. While it might be understandable that strong emotions should be aroused by certain claims or exhortations, *ad baculum* has nothing whatsoever to lend to the

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logic of an argument, and to claim (even implicitly) that it has is to commit this particular fallacy. A variant is *ad misericordium*, or the appeal to pity.

Ad hominem

To attack the personal reputation of a debating opponent is a well-known recognized manoeuvre in public debate. To do so is almost invariably fallacious, but there is one situation in which it might be valid. This is when the opponent's argument relies on some relevant claim about himself as one of its propositions. In such circumstances, the use of *ad hominem*, if based on relevant and true information about the speaker, can render his own argument worthless. A great favourite of politicians, though not always for the right reasons!

Ad populum

The appeal to popular opinion: another great favourite of politicians and hustlers ('The British people know . . .'). Invariably fallacious, as the claim that the speaker's point is already accepted by a great number of people can have no bearing on the validity of the argument. And of course, it is always possible that this great number of people, even if they do agree with the speaker, could be equally wrong.

Ad verecundium

The appeal to sapiential authority, expert knowledge, or wise judgement. Prone to notorious problems. The main errors: (1) irrelevance; (2) misattribution; (3) omissions of context; (4) inappropriateness; (5) lack of direct evidence; (6) favouring one authority over another (equally qualified) but dissenting authority; (7) ignoring relevant points of difference among expert authorities.

Ignoratio elenchi

The misconception of refutation—or simply confusing the issue with irrelevant claims. This might be why so many politicians in debate take so long to get to the point. At its clumsiest it consists simply of changing the subject. More sophisticated versions can be more difficult to detect, for example, the introduction of irrelevant or illegetimate qualifiers, e.g. one might assert that 'No *true* Scotsman would put ketchup on a haggis!', but such an invocation would neither disqualify Scotsmen who did so from being Scottish, nor the practice itself from being a legitimate habit of the Scots (although it might be disqualified on other grounds!). It is worth noting that this fallacy is not detected by most kinds of formal logic. *Ignoratio elenchi* can be combined with any of the preceding fallacies.

Petitio principii

The circular argument. In v*icious* circularity the conclusion has already appeared earlier in the argument (sometimes only implicitly); in *virtuous* circularity the claim is that as there is no better argument, this will have to do, but this does not make the logic any better. J.S. Mill argued that *all* deductive logic is infected with circularity! (1879) and (as noted above, this chapter) there is a sense in which mathematics is too. So what matters is whether the 'circle' is large enough, or sufficiently unobvious, to add anything useful to our understanding of the issues.

Post hoc ergo propter hoc

(Or simply *post hoc*) A complex family of related fallacies, to do with inferring an incorrect causal relation between two events. There are seven variations: (1) inferring a causal connection after just one instance; (2) inferring a causal connection on the basis of just one correlation with previous instances; (3) reversing cause and effect; (4) inferring a direct causal connection between two events that are connected only in that they share a common cause; (5) confusing causation and resemblance; (6) attributing cause to an (apparently) necessary (but not sufficient) condition; (7) ignoring inconvenient counter-instances or confounding data.

Affirming the consequent

This is an invalid inference from a conditional statement. Given 'If *p* then *q*', it is a fallacy to argue in reverse that because *q* then *p*. It is valid, however, to *deny the consequent*, arguing that, if *q* is false, *p* must also be false—this inference is licensed by the conditional, if true.

Denying the antecedent

Another invalid inference from a conditional statement. Given 'If *p* then *q*', it is a fallacy to argue that because *p* is false, *q* must also be false. It is valid, however, to affirm the antecedent, arguing that, if *p* then *q* (this is simply the inference licensed by the true conditional statement).

Equivocation

Relies on *semantic ambiguity* to allow a shift in the meaning or scope of a word between different parts of an argument. Whether it is fatal to the validity of the argument depends on how crucial the shift in meaning is. If the argument hangs on it, then it could be fatal. The danger of equivocation is greatest when *systematic ambiguity* exists (e.g. 'money', 'value'). This is a fallacy that is probably impossible to detect using formal logic.

Amphiboly

Relies on *grammatical ambiguity* to allow a shift between the grammatical status of a word, usually the shift is from an adjective to a verb or vice versa. Probably rare, if it exists at all (some writers claim it is not a true fallacy).

Illegitimate changes in the scope of a key operator

For example, negation or conjunction may range over variable domains in the same argument, but the conclusion relies on the scope of these operators being held constant. Easier to detect in formal logic than in informal logic—in fact, in the latter it can be almost impossible to detect.

Semantic vagueness

Closely related to equivocation, and the two can coexist. The point about semantic vagueness is that, if there is a lack of clarity or precision as to the meaning of key terms, then the same problem will infect any conclusion that is reached.

Inconsistency in a conjunction claimed as true

Where a proposition is actually a conjunction of propositions, the existence of inconsistency among this set will produce one or more contradictions; and, as a contradiction can never be true, the premiss will be false; and if the argument depends on the premiss being true, then the conclusion will be false.

The fallacy of composition

Concerns confusion over the transferability of valid forms of inference over different levels of composition. The reasoning that applies at the micro-level is confused with the reasoning that applies at the macro-level; when the confusion is downwards in scale, the label *fallacy of division* is sometimes used (e.g. macroeconomics versus microeconomics).

The fallacy of independence

Assuming in probabilistic calculations that successive outcomes are independent when they are not. It involves a faulty estimation of the conditional probability of each event—for independence to hold, conditional probability must be held constant.

Confusing prior and posterior probabilities

Assuming in probabilistic calculations that outcomes are dependent when they are not. Usually involves a misconception of randomness: randomness is uninfluenced by preceding outcomes, e.g. 'it comes up 'heads' this time, so it must be 'tails' next'.

Secundum quid

Assuming that one has sufficient information to make a generalization. Unfortunately, what is true in a certain respect (or under certain circumstances) may not be true in others. Faulty generalization of this kind is behind many errors of reasoning, but it is probably impossible to produce a heuristic for detecting it at work, because to do so would involve generating a method for revealing whether all the necessary qualifications have been made, and this is highly ambitious. And, of course, it takes only one counter-example to disprove any general claim. This fallacy lies behind many sampling errors in statistical analysis, two of the most serious being insufficient sample size, and sample bias.

Masked man fallacy

Arguing that knowledge of something under one description entails knowledge of it under another description; conversely, arguing that ignorance of something under one description entails ignorance of it under another description. For example, just because I know your identity but do not know the identity of the masked man, does not entail that you are not the masked man.

Quantifier shift fallacy

A fallacy involving a shift in the scope of quantifier terms; e.g. 'every girl loves some boy (or other), therefore there is some (one) boy whom every girl loves'. The shift in scope of the quantifier is possible only because it is ambiguous in ordinary language. In formal logic it is easier to detect. In symbolic form: $(\forall x)\ (\exists y)\ (Fxy)$ therefore $(\exists \boldsymbol{y})\ (\forall \boldsymbol{x})\ (Fxy)$.

The socratic fallacy

To argue that knowledge is founded on the ability to define the object(s) of knowledge. This fallacy is committed by the Socrates of the early Plato dialogues. It produces two paradoxes: first, it disqualifies from knowledge many who have every appearance of possessing it (in terms of intuitive ability); second, it disqualifies testing of definitions of the object of knowledge against known instances of that object. This fallacy is behind much of the faulty thinking about concepts of disorder that we covered in Chapters 2–4.

Reading guide

Introductions to logic

- Informal logic is more accessible than formal logic and is therefore a useful starting-point. A well established introduction is: Douglas Walton's *Informal Logic* (1989).
- The following more recent introductions to informal logic are particularly clear: Warburton, N. (2000) *Thinking from A to Z*, and Baggini, J. and Fosl, P. (2003) *The Philosopher's Toolkit: a compendium of philosophical concepts and methods.*
- A very readable introduction to the study of fallacies is Woods, J. & Walton, D. (1982) *Argument: the logic of the fallacies.*

Formal logic

- An accessible introduction to formal logic is Guttenplan, S. (1997) *The languages of logic: an introduction to formal logic.*
- Less readable but with many exercises is Lemmon, E.J. (1965) *Beginning Logic.*

Philosophy of logic

- Two highly readable treatments of the philosophy of logic (sometimes called philosophical logic) are: Haack, S. (1978) *Philosophy of Logics*, and Wolfram, S. (1989) *Philosophical Logic.*
- For a more advanced text, there is Simpson, R.L. (1988) *Essentials of Symbolic Logic.*

Truth

- An introductory level text on truth is Schmitt, F.F. (1995) *Truth: a primer*.
- More advanced is Kirkham, R.L. (1995) *Theories of Truth: a critical introduction*.
- For a critique of the very notion of truth, try: Allen, B. (1995) *Truth in Philosophy*.

Logical puzzles

- For those who like logical puzzles, there is Sainsbury, R.M. (1988) *Paradoxes*.

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